

ANALYSIS OF TEACHERS' QUESTIONING IN SUPPORTING MATHEMATICAL ARGUMENTATION BY INTEGRATING HABERMAS' RATIONALITY AND TOULMIN'S MODEL

Yuling Zhuang
University of Georgia
yuling.zhuang25@uga.edu

AnnaMarie Conner
University of Georgia
aconner@uga.edu

Teachers' questioning is a pivotal contributing factor to support students' engagement in productive mathematical collective argumentation. Following Habermas' (1998) construct of rational behavior, we attempted to demonstrate how teachers' questioning can be framed based on this construct. Adapting Habermas' construct with Toulmin's (1958/2003) model for argumentation, we conducted a case study to analyze how a prospective secondary teacher used questions to support collective argumentation in one of her student teaching classrooms. We also explored how the prospective teacher's interpretations of argumentation related to her use of rational questioning. The results suggested that this theoretical integration can be a useful tool to frame teacher questioning based on teachers' intentions and organizations of argumentation. Some educational implications for teacher professional development are provided.

Keywords: Classroom Discourse, Reasoning and Proof, Research Methods, Teacher Education- Preservice

Introduction

Purpose Statement

Developing mathematical reasoning is central to mathematics education. Considerable evidence suggests the beneficial effects of students articulating their mathematical reasoning and challenging others when engaged in collective mathematical argumentation (e.g., Krummheuer, 1995). *Collective mathematical argumentation* can be defined broadly as “any instance where students and teachers make a mathematical claim and provide evidence to support it” (Conner, Singletary, Smith, Wagner, & Francisco, 2014, p.44). Collective mathematical argumentation occurs in a mathematics class when the teacher and students (or a small group of students working independently) work together to construct or reject mathematical arguments.

Facilitating productive mathematical argumentation is challenging for many new teachers. As stated in *Principles to Actions: Ensuring Mathematics Success for All* (National Council of Teacher of Mathematics, 2014), effective mathematics teachers are expected to use purposeful questions to assess students' conceptual understanding and to advance students' reasoning and sense making about important mathematical ideas and relationships. Teachers' questioning has the potential to bring students into a conversation and promote participation, and the type and quality of questions can have significant impact on students' engagement in productive argumentation. By examining teachers' actions in support of student participation in mathematical argumentation, Conner et al. (2014) found that posing questions to elicit parts of arguments is a useful strategy that supports collective argumentation. Therefore, it is necessary to understand what factors influence teachers' questioning and how teachers use different types of questioning to support collective argumentation in mathematics classrooms.

Boero (2006) proposed that Habermas' (1998) construct of *rational behavior* can become a frame to deal with the complexity of discursive practices in the intersection of three kinds of rationality: epistemic (inherent in the control of validation of statements), teleological (inherent in the strategic choice of tools to achieve the goal of the activity), and communicative (inherent

in the conscious choice of suitable means to communicate understandably within a given community). Researchers have highlighted the importance of Habermas' (1998) construct as a tool to analyze didactical obstacles inherent in proving and argumentative activities (e.g., Boero, 2011; Boero & Planas, 2014). Under the vygotskian didactical perspective, Douek suggested using *rational questioning* as a method to "organize the mathematical discussion according to the three components of rationality" (Douek in Boero & Planas, 2014, p. 210). The teacher plays an essential role in creating a suitable context to promote the vygotskian dialectics and develop argumentation.

An interesting aspect of this construct is that Boero, Douek, Morselli, and Pedemonte (2010) illustrated the possible adaptation of combining Habermas' (1998) construct and Toulmin's (1958/2003) model for argumentation to study discursive practices related to proving and argumentative activities; they claimed that the combination of these two theoretical frameworks might provide a comprehensive frame that would allow one to better analyze students' proving and argumentative activities in mathematics classrooms and support teachers as they plan and carry out rational classroom interventions.

Research Questions

The goal of this study is to build on Habermas' (1998) construct of rational behavior, and to integrate Toulmin's (1958/2003) model for argumentation to ascertain the potential to use this tool to analyze teacher questioning through the lens of supporting mathematical collective argumentation. In addition, we are interested in how teachers' interpretations of argumentation relates to their pedagogical choices for rational questioning. The following research questions guided this study:

1. How does a prospective secondary teacher use rational questioning when guiding collective mathematical argumentation?
 - Which component of rationality is privileged in class?
 - What does rational questioning tell us about a teacher's support for argumentation in class?
 - What combinations of components of rational questioning support students' contribution of argument components?
2. How does a teacher's interpretations of argumentation align with his or her use of rational questioning?

Theoretical Framework

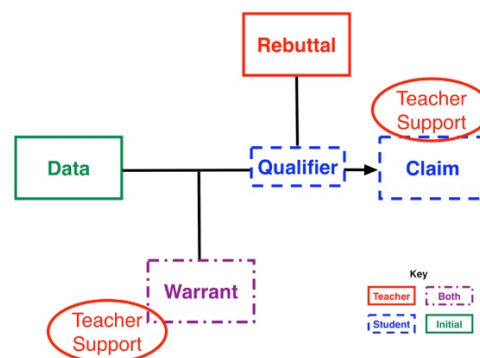
In order to make sense of classroom experiences, we need to broaden our interpretative stance by developing a sociological perspective on mathematical activity. For this purpose, in this study we employed the emergent perspective which emphasizes the idea that knowledge is created through actions and interactions, and that the development of an individual's reasoning and sense-making processes cannot be separate from his or her participation in the social interaction of taken-as-shared mathematical meanings (Cobb, Yackel & Wood, 1992).

According to Habermas' (1998) construct of rational behavior, we developed a Teacher Questioning Framework (see Table 1) by inferring the teacher's intentions and consciousness in asking the question. The framework consisting of three components of rationality and we defined *rational questioning* as a question that contains at least one component of rationality. At times, for clarity, we call a question epistemic rational questioning if it contains an epistemic rationality component.

Table 1: Teacher Questioning Framework

Component of Habermas' Rationality	Description of Questions	Examples
Epistemic Rationality	The questions used not only allow students to provide and evaluate arguments or ideas but also intend to challenge students by asking them to reason and make sense of their arguments and ideas.	Can you tell me why?
Teleological Rationality	The questions used allow students to show or reflect on the strategic choices that they used to achieve their arguments or ideas.	What do we need to do to the cards in the hat to make sure that every single family member has an equally likely chance of being drawn?
Communicative Rationality	The questions used allow students to communicate or reflect on the steps involved in their reasoning and arguments to ensure that their ideas can be understandable in the given community.	So will somebody raise their hand and tell me in their own words what they think it means to be equally likely?

In education literature, Toulmin's (1958/2003) model has been widely used to identify argument components (*claims, data, warrants, rebuttals and backing*) and analyze argumentative activities in classes (see Krummheuer, 1995 for more details on Toulmin's model). Our adaptation of Toulmin's diagrams (see Figure 1) includes the use of color and line style to record the contributor(s) of a component for a given argument and uses "Teacher Support" to denote teachers' contributions and actions (for this specific study, we focused only on teachers' questioning) that prompt or respond to parts of arguments (see Conner et al., 2014 for more details on the development of the framework).

**Figure 1.** Adaptation of Toulmin's (1958/2003) Diagram for an Argument

In this study, we adapted Habermas' (1998) construct of rational behavior to analyze the rationality of teachers' questioning and employed Toulmin's (1958/2003) model to examine how a teacher used these rational questioning to teaching.

Context and Methods

Context

This study is part of a larger project in which we are examining how teachers learn to support collective mathematical argumentation. The participant, Cathy (a pseudonym) was purposefully selected in this study based on her good understanding of argumentation as she participated in a unit on argumentation during her course work and be willing to support student engagement in argumentation during her student teaching. We observed Cathy's classroom teaching during a statistics and probability unit which focused on chance processes and probability models. The main data sources in this study included (1) video recordings and transcriptions of one representative day of Cathy's classroom teaching, (2) Cathy's interactions during a unit of instruction on collective argumentation (the argumentation unit) at the beginning of her pedagogy course in the mathematics teacher education program, and (3) a reflection that Cathy wrote about argumentation based on her observations in secondary classrooms.

Methods

We used Teacher Questioning Framework (see Table 1) to establish our starting codes to examine teachers' questioning in Cathy's class. A simple enumerative approach was also used to quantify verbal data in order to explicate the patterns that emerged from the open-coding process. Because we want to see how Cathy used her rational questioning to organize arguments, we used Toulmin's (1958/2003) model to diagram every episode of argumentation in Cathy's class, and we noted the parts of an argument that each question prompted. Finally, we used thematic analysis to understand Cathy's priorities and thoughts about argumentation based on her classroom discussions in the argumentation unit at the pedagogy course and her written reflection about argumentation a few weeks into the course.

Results

Use of Rational Questioning

According to our Teacher Questioning Framework (see Table 1), 66% (80/122) of Cathy's questions involved rational questioning. Among these rational questions, the largest component was communicative rationality (83%), followed by teleological rationality (81%); and the portion that involved epistemic rationality was smallest (40%). Although Douek's rational questioning seems to presuppose all three components of rationality, it is apparent that communicative rationality occupied a dominant position in Cathy's class. According to our analysis, only 34% (27/80) of the rational questioning contained all three components of rationality. Moreover, we found that Cathy used a variety of combined forms of rational questioning: some questions included two or three components of rationality and others only involved one.

Our diagrams of argumentation indicated that 59% (72/122) of Cathy's questions were asked during argumentative activities. Among these questions, most of them (81%) contained at least one component of rationality, which showed that Cathy had the intention to use rational questioning to support collective argumentation. Additionally, for these rational questions that were involved in argumentation, most of them had a communicative rationality component (85%) and a teleological rationality component (75%), but only 33% (19/58) had an epistemic rationality component. An interesting aspect of this result is that compared to the percentage of

each rationality component Cathy used in her whole class, as reported previously, we found that little difference existed for Cathy in her use of rational questioning during argumentative activities and non-argumentative activities.

According to the above results, Cathy showed a strong emphasis on developing her students' communicative rationality. Some of her communicative rational questioning asked students to explain their reasoning steps (e.g., Can you explain a little bit?). Some of them asked students to communicate ideas more clearly (e.g., And which numbers are they?) or to rephrase a mathematical definition to ensure that their explanations were understandable in the given community (e.g., What does it mean for a coin to be fair?). Cathy also used communicative rational questioning to encourage more students to engage in collective argumentation; this process was evidenced by Cathy often waiting a couple of minutes to see more students' raise their hands before she asked a student to answer her question. In addition, Cathy also pushed her students to evaluate and make sense of the claims and warrants that were provided by other students (e.g., Does it make sense why these two are counted separately?). According to our diagrams of argumentation, for rational questioning containing a communicative component, 24 (out of 44) prompted claims and data/claims while only 13 (out of 44) prompted warrants, which means that Cathy mainly used communicative rational questioning to promote student participation in the construction of arguments.

Within the rational questioning that contained a teleological component in Cathy's class, we found that most all of these questions were strategically goal-oriented reasoning: Cathy first let the students find the possible results of an event, and she next asked them to conjecture whether these events had equally likely chances of happening. Sometimes she asked students to justify their answers by using the classical approach to probability. On a few occasions, Cathy wanted to highlight the importance of the strategic choice of using examples, and she asked her students to provide different examples of what it meant to be "equally likely": "Is any, can anybody think of a different example? That might give you [an] equally likely [chance]?"

Cathy used most of her epistemic rational questioning to push her students to justify why their arguments hold. For example, she asked, "Would you mind sharing your reasoning with the class?" However, only 33% of the rational questioning that contained epistemic components. Cathy's goal for students was apparently less focused on epistemic rationality than on teleological and communicative rationality. This difference was also evidenced by our diagrams of argumentation, within which only 17 (out of 72) questions prompted warrants while 45 (out of 72) prompted claims and data/claims.

As mentioned above, some rational questions involved all three components of rationality (e.g., Can you explain to me why it's going to be fifty fifty?), and we found that more than half (56%) of these questions helped students contribute warrants, which means these questions provided students opportunities to reason about and make sense of their arguments — one of the most important aspects of argumentation for students. Some of Cathy's rational questions contained only teleological and communicative components (e.g., Can somebody remind me how I find the volume of any prism? I will wait until I see a few more hands.), and among these questions, 11 (out of 19) prompted students' claims and data/claims. Cathy used this kind of rational questioning to ask students to communicate their arguments and to focus on the strategic tools that they used to solve the problem.

Interpretations of Argumentation

By examining portions of classroom discussions during the argumentation unit that Cathy participated in and her written reflections about argumentation, we discovered two clear themes.

First, Cathy regarded the role of teachers as asking questions and calling on students to participate. For example, during an in-class discussion in the argumentation unit, Cathy stated, “The role of the teacher is to facilitate, asking questions, and choosing students to participate...” Another theme that emerged concerned Cathy’s desire that students be the main contributors to the arguments. In her written reflection, she explained that an effective argument should involve multiple students, and she labeled as an ineffective argument as one that involved only one student.

Our analysis suggests that Cathy’s interpretations of argumentation align with her use of rational questioning to support collective argumentation during her student teaching classroom practice. Most of the rational questioning that Cathy asked contained a communicative rationality component, which aimed to bring students’ voices into the discussion and encouraged more students to participate in collective argumentation. Data from Cathy’s student teaching showed that she used communicative rational questioning not only to encourage students to communicate their ideas more clearly but also to provide opportunities for her students to lead the classroom discussion. For example, she would ask questions such as the following: “Can I have a volunteer go to the board and help us find the unit rate?” Several times after a student gave a claim and warrant, Cathy used communicative rational questioning — “Does anybody have any questions for Ashley?” — to intentionally slow down the presentation to make sure it made sense to others and to inspire students to critique one another’s arguments. The large number of communicative rational questions and the way Cathy used them throughout her student teaching seemed consistent with her belief that the teacher was responsible for promoting student engagement. Cathy also used teleological and epistemic rationality questioning to support of her idea of the teacher as a facilitator and used teleological rationality in her questions to strategically develop goal-oriented reasoning, and then followed up with epistemic rational questioning to request further explanations from her students.

In addition, we noticed that 34% (42/122) of the questions were non-rational, which could be categorized into two types of questions. One type of non-rational questions was either/or questions that were situated in argumentative activities, which prompted claims and asked students to construct results for the final step of argumentation. An example of this type of question would be as follows: Is it equally likely or not equally likely? Another type of non-rational questions was used to present information. For example, Cathy often asked students to read the task. She also asked several factual questions in order to create a probability problem context. We labeled these types of questions as contextual questioning, which may be specific to teaching probability. The way in which Cathy used non-rational questioning showed that instead of giving all the information herself, she asked students to provide data and final claims in the arguments, which aligned with her goal of promoting participation among the students.

Discussion

Many researchers have used Habermas’ (1998) construct to conduct studies that centered on *students* (e.g., Morselli & Boero, 2011), and our study showed that this theoretical framework can also be used to investigate *teachers’* intentions when they ask specific kinds of questions. Some researchers have categorized types of teachers’ questioning within classroom discourse (e.g., Wood, 1998). The integration of Habermas’ construct with Toulmin’s (1958/2003) model as a tool to analyze teachers’ questioning provides us with a more comprehensive perspective for understanding the roles of teacher questioning within collective argumentation. The two models complement each other in the following sense: Habermas’ lens help us to identify fine grained rationality components of teachers’ questioning and also how teachers’ questioning is

constrained in relation to the three components of rational behavior; the teacher then using rational questioning to control the fundamental steps of argumentation is seen through Toulmin's lens. We see the function of rational questioning as fully implementing an approach that pushes students to make sense of arguments, to strategically choose effective tools or methods in problem-solving, and to communicate understandably in a given community, which are essential goals of collective argumentation. This study provides empirical evidence to support Boero et al.'s (2010) idea about the value of integrating Habermas' elaboration of rational behavior with Toulmin's model for argumentation to deal with the complexity of learning and teaching activities, particularly with respect to proving and argumentative activities.

Douek, in Boero and Planas (2014) suggested that teacher's rational questioning can support students' "rationalization" of discourse and develop mathematical discussions along the three components of rationality. Based on our analysis, most of the questions that were posed by Cathy to support collective argumentation were connected to the three components of rationality. Moreover, in this study we found that Cathy used different combinations of rational questioning in her class, and most of them involved only one or two rationality component. According to our analysis, these kinds of rational questioning can also support collective argumentation, and the function of different kinds of rational questioning may vary across different classroom activities. More research is needed to explore whether any other forms of combinations can support argumentation and when are the best times and contexts for teachers to use them.

Although the study showed that Cathy's rational questioning included all three components of rationality, we found that the epistemic component made up a relatively small portion. This result raises an important question: Why do some components of rationality occur less frequently than others? One reasonable explanation from our study is that teachers attempt to use different kinds of rational questioning based on their interpretations of argumentation during course work. We also considered that a teacher's use of different kinds of rational questioning may depend on different mathematical domains, content, and various contexts in the classroom (e.g. whole class discussions or small group discussions). In this study, we only analyzed one of Cathy's probability classes, and more studies are needed to explore what factors influence a teacher's intentional choices of different kinds of rational questioning.

Implications

Our results indicated that Cathy was primarily concerned about her students' participation and thought that the role of teacher was a facilitator; these themes remained consistent with the way she adopted rational questioning during her student teaching. Cathy seemed to take her interpretations of argumentation into her student teaching placement, and it is reasonable to infer that the argumentation unit in her pedagogy course may have influenced her use of questioning for instruction. Our results also suggest that teacher education programs should demonstrate the power of collective argumentation and promote teachers' understanding of the roles of a teacher in questioning.

As mentioned above, Cathy used almost the same proportion of different kinds of rational questioning when she led argumentative activities and non-argumentative activities, which showed that Cathy's intention to strategically use rational questioning as a tool to support collective argumentation spilled over into her regular instruction. More research is necessary to examine how to use the three components of rationality in questions at the moment when argumentation does not develop or when argumentation is begun but breaks off. We hypothesize that understanding specific components of rational questioning could help teachers to integrate multiple questioning strategies to support collective argumentation in a specific content and in a

specific context. Future studies should continue to analyze the benefits of rational questioning and how can it be incorporated into teacher education program to make it more approachable to prospective teachers.

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